

Hypocoercivity without changing the scalar product

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Hypocoercive methods allow to prove the longtime convergence of the law of degenerate stochastic processes such as Langevin dynamics or random time HMC. From a technical viewpoint, these approaches usually involve a change of scalar product (as in the H^1 approach à la Villani, or the L^2 approach of Hérau and Dolbeault/Mouhot/Schmeiser). I will discuss two approaches which do not require such a change of scalar product, and henceforth provide the current sharpest quantitative estimates on the resolvent of the generator, which makes it possible for instance to bound the variance of estimators based on time averages along a realization of the process. The first approach, originally due to Armstrong/Mourrat, was adapted to dynamics on the whole space by Cao/Lu/Wang, and is based on space-time Poincaré inequalities. I will review some progress made with Giovanni Brigati on better understanding the algebraic structures behind this method, and possibly mention extensions to weak/weighted Poincaré inequalities (in collaboration with Lihan Wang and Andi Wang). The second approach, developed with Etienne Bernard, Max Fathi and Antoine Levitt, is based on Schur complements and provides direct bounds on the resolvent using the specific saddle-point like structure of the generator under an appropriate orthogonal decomposition of L^2 .