Elliptic and hypoelliptic deformations of Hodge theory and analytic torsion

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This 6h lecture, presented by F. Nier, was prepared with S. Shen who cannot finally attend the conference. It will show in several steps how Witten and Bismut deformations of Hodge theory are used for the study of analytic torsion. Below is the outline.

1. Algebraic Hodge theory and supersymmetry: a finite dimensional algebraic approach; presentation and classical examples.

2. Analytic torsion. Motivations, definition and meromorphy of the zeta function associated with some semigroup generators. Parametric analysis of the analytic torsion.

3. Lie derivatives for hamiltonian vector fields viewed as "Hodge operators". The hypoelliptic Laplacian and functional analysis results about it.

4. Some details about Ray-Singer metric.