Semiclassical hypocoercivity and Eyring-Kramers law for degenerated Fokker-Planck operators

Loïs Delande

Institut de Mathématiques de Bordeaux

We investigate certain Fokker-Planck operators and the Witten Laplacian in the low temperature regime, considering potentials that are not necessarily Morse. Our main focus is on the spectral behavior near zero of the associated operators, for which we aim to provide a precise characterization. Such a spectral description allows us to derive detailed insights into the long-time dynamics of the solutions, including quantitative results on return to equilibrium and metastability.

We begin with the analysis of the Witten Laplacian, a selfadjoint operator. Our approach involves adapting recent quasimode constructions to our non-Morse setting. Under a generic assumption on the degenerate potential, we successfully derive the desired spectral description.

Next, we turn to the Fokker-Planck operator with generalized degenerate coefficients. Here, degeneracy refers to the fact that the microlocal symbol of the operator is no longer locally quadratic. Leveraging the results obtained for the Witten Laplacian, we address the analytical challenges introduced by this degeneracy. Our strategy partly relies on resolvent estimates derived via hypocoercive techniques. As a result, we are once again able to establish an Eyring-Kramers-type formula for the spectrum of this operator.