Analytic hypoellipticity in condensed matter physics

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Twisted bilayer graphene (TBG) is a setting of a remarkable theoretical (Bistritzer–MacDonald, 2011) and experimental (Cao et al 2018) discovery, that two sheets of graphene twisted by certain (magic) angles display unusual electronic properties such as superconductivity. The key in the mathematical treatment of this is the existence of protected states in the so-called chiral model (Tarnopolsky–Kruchkov–Vishwanath, 2019).

In this talk I will explain exponential decay (as the angle of twisting goes to zero) of these states near the hexagon spanned by the stacking points (points of high symmetry). Near interior points of the edges it follows from general results (joint with M Hitrik) and based on the geometry of Poisson brackets (with a different proof recently provided by J Sjostrand). Near the stacking points (vertices of the hexagon) it follows from an analytic hypoellipticity argument based on the specific structure of the operator (joint work with Z Tao).

I will also present the difficulties in proving such decay at the center of the hexagon where the operator is no longer of principal type.