

Small eigenvalues of non-reversible metastable diffusion processes with Neumann boundary conditions

Let $\Omega \subset \mathbb{R}^d$ be a bounded smooth domain and $b: \Omega \rightarrow \mathbb{R}^d$ be a smooth vector field. We focus on the associated overdamped Langevin equation :

$$\dot{X}_t = b(X_t) + \sqrt{h}\dot{B}_t$$

in the low regime temperature where $h \rightarrow 0$ and in the case where b admits the decomposition $b = -\nabla f - \ell$ with :

- ℓ a smooth vector field;
- f a Morse function on $\overline{\Omega}$ admitting several local minima;
- $\nabla f \cdot \ell = 0$ on $\overline{\Omega}$.

In this framework, minima of the function f correspond to metastable states for this Langevin dynamics. In this context, we analyse the spectrum of the infinitesimal generator of the dynamics :

$$L_h = -\frac{h}{2}\Delta + \nabla f \cdot \nabla + \ell \cdot \nabla$$

with Neumann boundary conditions. In this case, moving particles will remain trapped inside the domain. More specifically, we will consider additional hypotheses ensuring that the measure $e^{-\frac{2f}{h}} dx$ is invariant.